Discrete Random Variables

4.2 Expected Value: The Mean of a Discrete Random Variable

1. A pair of six-sided die is rolled four times. We record the number of sevens rolled over the four rolls. Determine if the results form a probability distribution. If so, find the expected value of the resulting probability distribution.

X	P(X)
0	0.482
1	0.386
2	0.116
3	0.015
4	0.001

2. Converting the above probability distribution into a game, players can risk $4 to bet that a seven will not be rolled in four throws of the dice. If the player wins, he will be paid $7 (a three-dollar profit), but if the player loses he/she loses his/her $4 bet. What is the expected value for this game? Is this a smart bet to make in the long run?

3. The Florida Lotto requires you to pick six numbers from 1 to 53. If you win by having all six of the lotto numbers on your ticket, you will receive $2,000,000. If you lose, you will lose your dollar. What is the expected value on a Florida Lotto ticket? How do you interpret this number? (Note: If you use combinations, you will see that the probability of winning is 1/22,957,480)

Answers:

1. It is a probability distribution since the probabilities add to one. The expected value is $E(X) = 0.667$.

2. The probability he rolls at least one seven is 0.386 + 0.116 + 0.015+ 0.001 =0.518. The probability he doesn't roll a seven is 0.482. The expected value is then $E(X) = 3(0.482) - 4(0.518) = -0.626 \approx -\$0.63$ or a 63 cents loss for every bet made. This is not a good bet in the long run.

3. Here is how the probability table looks:

	X	P(X)	X*P(X)
Win	+1,999,999	0.0000000436	0.0871175364
Lose	-1	0.9999999564	-0.9999999564
			≈-0.91

This means if we played lottery an infinite number of times (or at least a great many times) we would win sometimes, but not nearly enough to make up for the amount we would lose over that time. Our expected value is a 91 cents loss for every dollar spent on lotto. This is not a smart bet, but if a person doesn't go overboard, it can provide a cheap form of entertainment for people not morally opposed to gambling.